Whitehead’s Problem and Reverse Mathematics

杨森

Whitehead's Problem asks whether every Whitehead group is free. Stein in 1951 showed that every countable Whitehead group is free and Shelah in 1974 showed that Whitehead's Problem in general is independent of ZFC. In this paper, we study Whitehead's Problem in reverse mathematics and in particular, the proof-theoretic strength of Stein's Theorem. We established that over the base theory WKL_0, ACA_0 is equivalent to Stein's theorem; whereas over the base theory RCA_0, WKL_0 and Stein's theorem are incomparable.