Possibilities and Actuality

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2024/04/27

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This question is reasonable since modern philosophy is littered with ‘possible-world’ talks.

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‘Possible worlds semantics’ is a little different.

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$$W(\varphi) := \Diamond\varphi \wedge \forall r(\Box(\varphi \rightarrow r) \vee \Box(\varphi \rightarrow \neg r))$$

$$\text{(WP)} \quad \forall p(\Diamond p \rightarrow \exists q(W(q) \wedge \Box(q \rightarrow p)))$$

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- present a argument for (WP);
- point out where we don't like it;
- present a formal model in which (WP) is false but also supports a reasonable theory.

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- Certainly the actual world exists. From Humberstone's *From Worlds to Possibilities*:

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- Then this is a necessary truth as we gave it a logical argument.

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Since the above is an argument from logic, the conclusion can be necessitated, and we get $\Box\exists p(p \wedge W(p))$.

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Consequence: embrace (Free Logic) and (Propositional Contingentism).

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There is no defense, but there is a cool model.

An $\mathsf{S5}\Pi^{-}_{\mathsf{free}} + (\mathsf{WP}_{\Diamond})$ model

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- If we allow plural variables, then not all formulas express existing propositions. But otherwise yes.

Thanks!