Bilateral Gradual Semantics for Weighted Argumentation

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Outline

- Background
- Method
- Principles
- Semantics
- Conclusion

Abstract Argumentation is a reasoning model for evaluating arguments.

Dung, P.M. 1995. On the acceptability of arguments and its fundamental role in non-monotonic reasoning, logic programming, and n-person games. *Artificial Intelligence*

Abstract Argumentation

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Abstract the argumentation scenario as a directed graph:

- nodes for arguments
- arrows for attack relation between arguments

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Argumentation Graph

Definition (Dung 1995)

An *argumentation graph* is a pair $\langle \mathcal{A}, \mathcal{R} \rangle$, where $\mathcal{A}$ is a finite set of arguments, and $\mathcal{R} \subseteq \mathcal{A} \times \mathcal{A}$ is a finite set of attack relation.

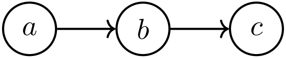

Figure: a attacks b and b attacks c

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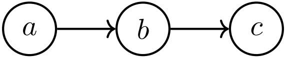

Figure: a attacks b and b attacks c

Evaluating the status of arguments is a central topic.

Argumentation Semantics

Semantics: criteria for evaluating the status of arguments

Baroni, et al. 2011. An introduction to argumentation semantics. *Knowledge Engineering Review*

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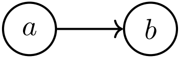

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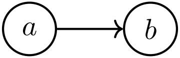

$$a \text{ kills } b \Rightarrow \begin{cases} a & \text{(survived)} \\ b & \text{(died)} \end{cases}$$

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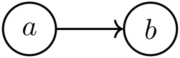
Semantics: criteria for evaluating the status of arguments



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accept $\{a\}$, reject $\{b\}$

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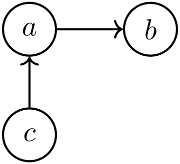
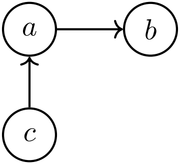
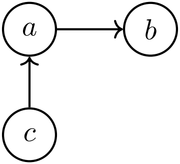

a
b
c

Argumentation Semantics



$$c \text{ kills } a \Rightarrow \begin{cases} \{b, c\} & \text{(survived)} \\ a & \text{(died)} \end{cases}$$

Argumentation Semantics



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accept $\{b, c\}$, reject $\{a\}$

Weighted Argumentation is designed to quantify the uncertainty in real-world argumentation.

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Definition

A *weighted argumentation graph* (WAG) is a triple $\langle \mathcal{A}, w, \mathcal{R} \rangle$, where $\mathcal{A}$ is a finite set of arguments, w a function from $\mathcal{A}$ to $[0, 1]$, and $\mathcal{R} \subseteq \mathcal{A} \times \mathcal{A}$ an attack relation.

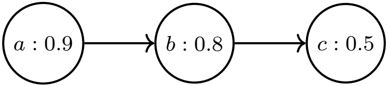

Figure: Arguments with basic weights

Question: How to evaluate arguments in WAG?

Gradual Semantics

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Gradual Semantics (Amgoud et al. 2017)

- Assign each argument an acceptability degree as its strength

A *gradual semantics* is a function $\mathcal{S}$ transforming any WAG to a measure function $Deg : \mathcal{A} \rightarrow [0, 1]$. $\forall a \in \mathcal{A}$, $Deg(a)$ is called the *acceptability degree* of a.

Amgoud, L. et al. 2017. Acceptability semantics for weighted argumentation frameworks. *IJCAI*

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The higher the acceptability degree, the stronger the argument.

Amgoud, L. et al. 2017. Acceptability semantics for weighted argumentation frameworks. *IJCAI*

Motivation

Argument strength of *positivity* and *negativity* should be separately considered in the evaluative process.

Cacioppo, J. et al. 1997. Beyond bipolar conceptualizations and measures: The case of attitudes and evaluative space. *Personality and Social Psychology Review*

Bilateral Gradual Semantics

Motivation

Argument strength of *positivity* and *negativity* should be separately considered in the evaluative process.

Idea: Enhance gradual semantics by incorporating the notion of *rejectability degree*.

Cacioppo, J. et al. 1997. Beyond bipolar conceptualizations and measures: The case of attitudes and evaluative space. *Personality and Social Psychology Review*

Bilateral Gradual Semantics

Motivation

Argument strength of *positivity* and *negativity* should be separately considered in the evaluative process.

Idea: Enhance gradual semantics by incorporating the notion of *rejectability degree*.

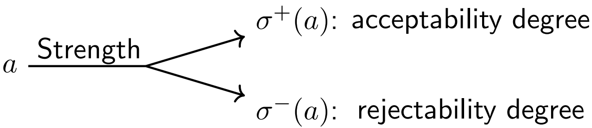
Example: Politicians may prefer safer arguments that receive less attack (i.e., with a lower rejectability degree).

Cacioppo, J. et al. 1997. Beyond bipolar conceptualizations and measures: The case of attitudes and evaluative space. *Personality and Social Psychology Review*

Bilateral Gradual Semantics

Definition

A Bilateral Gradual Semantics (BGS) $\mathcal{S}$ transforms any WAG $\mathbf{G} = \langle \mathcal{A}, w, \mathcal{R} \rangle$ to a function $Deg^{\mathcal{S}}_{\mathbf{G}}\colon \mathcal{A} \to [0,1] \times [0,1]$. For any $a \in \mathcal{A}$, $Deg^{\mathcal{S}}_{\mathbf{G}}(a) = (\sigma^{+}(a), \sigma^{-}(a))$ where $\sigma^{+}(a)$ and $\sigma^{-}(a)$ represent the acceptability and rejectability degree of a respectively.



How to define a reasonable BGS?

How to define a reasonable BGS?

Principle-based Approach

- Clarify the *source* that influence argument strength
- Establish *principles* that semantics should satisfy
- Define *well-behaved* semantics that satisfy the principles

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van der Torre, L.; and Vesic, S. 2017. The principle-based approach to abstract argumentation semantics. *Handbook of formal argumentation*

Bilateral Gradual Semantics

<table>
<tr><th>Degree</th><th>Source of Strength</th></tr>
<tr><td>acceptability</td><td>• basic weight
• acceptability degree of attackers
• rejectability degree of attackers</td></tr>
<tr><td>rejectability</td><td>• acceptability degree of attackers</td></tr>
</table>

Table: Non-reciprocity of Argument Strength in BGS

- Serve for the construction of various $Deg^{\mathcal{S}}_{\mathbf{G}}$
- Well-suited for a wide range of principles

BGS Principles

Principles represent desirable properties that a semantics may need to satisfy in practical applications.

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Basic items	Anonymity	Independence	Directionality
	Equivalence	Resilience	Proportionality
Symmetric	A-Neutrality	A-Weakening	A-Counting
	R-Neutrality	R-Strengthening	R-Counting
	A-Reinforcement	A-Weakening Soundness	A-Maximality
	R-Reinforcement	R-Strengthening Soundness	R-Minimality
Defense	Weakened Defense	Strict Weakened Defense	
Strategies	Quality Precedence	Cardinality Precedence	Compensation

Table: Priciples for Bilateral Gradual Semantics

BGS Principles

Example

- Anonymity: strength should not be impacted by its identity
- A-Counting: acceptability degree decreases as attackers increase
- R-Counting: rejectability degree increases as attackers increase
- A-Maximality: the acceptability degree of a non-attacked argument equals its basic weight.
- R-Minimality: the rejectability degree of non-attacked arguments equals 0.

Three Strategies

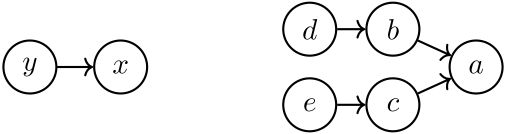
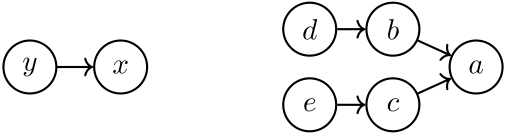


Figure: The argument x has a strong attacker, whereas the argument a has two weak attackers (each one is attacked). Which aspect is more significant: Quality or Quantity?



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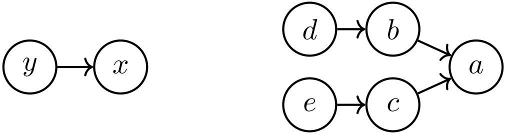

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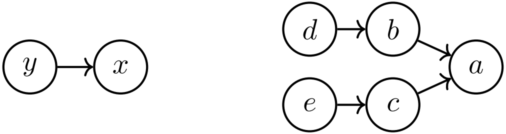
Three strategies:

- Quality Precedence (QP): the strongest attacker

Three Strategies



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Three strategies:

- Quality Precedence (QP): the strongest attacker
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Three Strategies

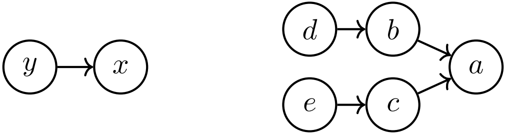

Figure: The argument x has a strong attacker, whereas the argument a has two weak attackers (each one is attacked). Which aspect is more significant: Quality or Quantity?

Three strategies:

- Quality Precedence (QP): the strongest attacker
- Cardinality Precedence (CP): the number of attackers
- Compensation: both quality and quantity of attackers

Links between Principles

Incompatible Results

1. If a semantics satisfies A-maximality, then QP, CP and Compensation are pairwise incompatible.
2. CP (resp. Compensation) is compatible with all basic principles.
3. A semantics cannot satisfy QP and all basic principles.

Links between Principles

Implications between Principles

1. {Directionality, Independence, A-Maximality, A-Neutrality} imply {A-Weakening Soundness}
2. {Directionality, Independence, R-Minimality, R-Neutrality} imply {R-Strengthening Soundness}
3. {Directionality, Independence, Equivalence, A-Maximality, A-Neutrality, A-Reinforcement} imply {A-Counting, A-Weakening}
4. {Directionality, Independence, Equivalence, A-Maximality, R-Neutrality, R-Reinforcement} imply {R-Counting, R-Strengthening}

BGS Iterative Functions

The best way of defining semantics for WAG is to introduce iterative functions that take any WAG as inputs and produce sequences of values that eventually converge.

Gabbay, D. and Rodrigues, O. 2015. Equilibrium states in numerical argumentation networks. *Logica Universalis*

BGS Iterative Functions

The best way of defining semantics for WAG is to introduce iterative functions that take any WAG as inputs and produce sequences of values that eventually converge.

Defining converged iterative functions is challenging.

- Restricted to acyclic graphs.
- Convergence is a conjecture.

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BGS Iterative Functions

h-Categorizor function (Hunter and Besnard, 2001)

$$f^{i+1}(a) = \frac{1}{1 + \sum_{b \to a} f^i(b)}$$

- Always converges over any input graph (Pu et al. 2014)

Besnard, P. and Hunter, A. 2001. A logic-based theory of deductive arguments. *Artificial Intelligence*

BGS Iterative Functions

AR-max-based function (Quality Precedence)	$f^{i+1}(a) = \frac{w(a)}{1+\max_{b\in Att(a)} \frac{f^i(b)}{1+g^i(b)}}$ $g^{i+1}(a) = \frac{\max_{b\in Att(a)} f^i(b)}{1+\max_{b\in Att(a)} f^i(b)}$
AR-card-based function (Cardinality Precedence)	$f^{i+1}(a) = \frac{w(a)}{1+\lvert Att^*(a)\rvert+\frac{1}{n}\cdot\sum_{b\in Att^*(a)} \frac{f^i(b)}{1+g^i(b)}}$ $g^{i+1}(a) = \frac{\lvert Att^*(a)\rvert+\frac{1}{n}\cdot\sum_{b\in Att^*(a)} f^i(b)}{1+\lvert Att^*(a)\rvert+\frac{1}{n}\cdot\sum_{b\in Att^*(a)} f^i(b)}$
AR-hybrid-based function (Compensation)	$f^{i+1}(a) = \frac{w(a)}{1+\lvert Att^*(a)\rvert+\sum_{b\in Att^*(a)} \frac{f^i(b)}{1+g^i(b)}}$ $g^{i+1}(a) = \frac{\lvert Att^*(a)\rvert+\sum_{b\in Att^*(a)} f^i(b)}{1+\lvert Att^*(a)\rvert+\sum_{b\in Att^*(a)} f^i(b)}$

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Main Theorem (Wang and Shen, 2024)

The above iterative functions always converge as i approaches ∞.

Proof Sketch

Assume an enumeration for arguments: $\mathcal{A} = \{a_1, ..., a_n\}$
$f^i(\mathcal{A})$: $(f^i(a_1), ..., f^i(a_n))$ $g^i(\mathcal{A})$: $(g^i(a_1), ..., g^i(a_n))$

- The sequences $\{f^{2i}(\mathcal{A})\}_{i\in\mathbb{N}}$ and $\{g^{2i+1}(\mathcal{A})\}_{i\in\mathbb{N}}$ are monotonically non-increasing and bounded by $(0, ..., 0)$.
- The sequences $\{f^{2i+1}(\mathcal{A})\}_{i\in\mathbb{N}}$ and $\{g^{2i}(\mathcal{A})\}_{i\in\mathbb{N}}$ are monotonically non-decreasing and bounded by $(1, ..., 1)$.

According to the Monotone Convergence Theorem,

$$\overline{f}(\mathcal{A}) = \lim_{i\to\infty} f^{2i}(\mathcal{A}),\ \underline{f}(\mathcal{A}) = \lim_{i\to\infty} f^{2i+1}(\mathcal{A})$$
$$\overline{g}(\mathcal{A}) = \lim_{i\to\infty} g^{2i+1}(\mathcal{A}),\ \underline{g}(\mathcal{A}) = \lim_{i\to\infty} g^{2i}(\mathcal{A})$$

Finally, we prove that

$$\underline{f}(\mathcal{A}) = \overline{f}(\mathcal{A}) \text{ and } \underline{g}(\mathcal{A}) = \overline{g}(\mathcal{A}).$$

AR-Max-Based BGS

The *AR-max-based semantics* ARM transforms $\mathbf{G} = \langle \mathcal{A}, w, R \rangle$ to $Deg_{\mathbf{G}}^{ARM}$, s.t. $\forall a \in \mathcal{A}$, $Deg_{\mathbf{G}}^{ARM}(a) = (\sigma^+(a), \sigma^-(a))$ with $\sigma^+(a) = \lim_{i \to \infty} f^i(a)$, $\sigma^-(a) = \lim_{i \to \infty} g^i(a)$.

By definition, we can show

$$\sigma^+(a) = \frac{w(a)}{1 + \max_{b \in Att(a)} \frac{\sigma^+(b)}{1+\sigma^-(b)}},$$

$$\sigma^-(a) = \frac{\max_{b \in Att(a)} \sigma^+(b)}{1 + \max_{b \in Att(a)} \sigma^+(b)}.$$

AR-Card-Based BGS

The *AR-card-based semantics* ARC transforms $\mathbf{G} = \langle \mathcal{A}, w, R \rangle$ to $Deg_{\mathbf{G}}^{ARC}$, s.t. $\forall a \in \mathcal{A}$, $Deg_{\mathbf{G}}^{ARC}(a) = (\sigma^+(a), \sigma^-(a))$ with $\sigma^+(a) = \lim_{i \to \infty} f^i(a)$, $\sigma^-(a) = \lim_{i \to \infty} g^i(a)$.

By definition, we can show

$$\sigma^+(a) = \frac{w(a)}{1 + |Att^*(a)| + \frac{1}{n} \cdot \sum_{b \in Att^*(a)} \frac{\sigma^+(b)}{1+\sigma^-(b)}}$$

$$\sigma^-(a) = \frac{|Att^*(a)| + \frac{1}{n} \cdot \sum_{b \in Att^*(a)} \sigma^+(b)}{1 + |Att^*(a)| + \frac{1}{n} \cdot \sum_{b \in Att^*(a)} \sigma^+(b)}.$$

Notice that $\frac{1}{n} \cdot \sum_{b \in Att^*(a)} \frac{\sigma^+(b)}{1+\sigma^-(b)} < 1$.

AR-Hybrid-Based BGS

The *AR-hybrid-based semantics* ARH transforms $\mathbf{G} = \langle \mathcal{A}, w, R \rangle$ to $Deg_{\mathbf{G}}^{ARH}$, s.t. $\forall a \in \mathcal{A}$, $Deg_{\mathbf{G}}^{ARH}(a) = (\sigma^+(a), \sigma^-(a))$ with $\sigma^+(a) = \lim_{i \to \infty} f^i(a)$, $\sigma^-(a) = \lim_{i \to \infty} g^i(a)$.

By definition, we can show

$$\sigma^+(a) = \frac{w(a)}{1 + |Att^*(a)| + \sum_{b \in Att^*(a)} \frac{\sigma^+(b)}{1+\sigma^-(b)}}$$

$$\sigma^-(a) = \frac{|Att^*(a)| + \sum_{b \in Att^*(a)} \sigma^+(b)}{1 + |Att^*(a)| + \sum_{b \in Att^*(a)} \sigma^+(b)}.$$

Principles satisfied by BGS

	ARM	ARC	ARH
Anonymity	✓	✓	✓
Independence	✓	✓	✓
Directionality	✓	✓	✓
Equivalence	✓	✓	✓
Resilience	✓	✓	✓
Proportionality	✓	✓	✓
A-Neutrality	✓	✓	✓
R-Neutrality	✓	✓	✓
A-Maximality	✓	✓	✓
R-Minimality	✓	✓	✓
A-Weakening	✓	✓	✓
R-Strengthening	✓	✓	✓
A-Weakening soundness	✓	✓	✓
R-Strengthening soundness	✓	✓	✓
A-Counting	-	✓	✓
R-Counting	-	✓	✓
A-Reinforcement	-	✓	✓
R-Reinforcement	-	✓	✓
Weakened Defense	✓	✓	✓
Strict Weakened Defense	-	✓	✓
Quality Precedence	✓	-	-
Cardinality Precedence	-	✓	-
Compensation	-	-	✓

Conclusion

- Evaluate argument through a bilateral perspective
  - Acceptability Degree + Rejectability Degree
  - Non-reciprocal Integration
- Establish principles for BGS
  - Desirable properties under the bilateral setting
  - Links between BGS principles
- Provide three well-behaved semantics
  - Quality Precedence, Cardinality Precedence and Compensation
  - Argument strength defined as limits of iterative sequences

Future Work

Extend BGS for Weighted Bipolar Argumentation Graphs

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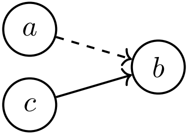

Figure: Solid arrow for attack relation; dashed arrow for support relation

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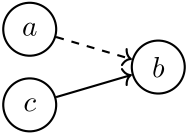
Extend BGS for Weighted Bipolar Argumentation Graphs



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The study of converged iterative functions in weighted bipolar argumentation graphs is still open, and our method shows significant potential for this topic.

Reference

[1] Dung, P.M. 1995. On the acceptability of arguments and its fundamental role in nonmonotonic reasoning, logic programming and n-person games. *Artificial intelligence*

[2] Amgoud, L.; Ben-Naim, J.; Doder, D.; and Vesic, S. 2017. Acceptability Semantics for Weighted Argumentation Frameworks. *IJCAI 17'*

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Thanks for your attention!