To do something else

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Outline

An old idea

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Anderson [1967] and Kanger [1971] followed this idea and independently developed a deontic logic. This is a classical modal logic and it leads to quite many problems.

A dynamic deontic logic

Starting from the same idea, Meyer [1988] proposed a deontic logic based on a fragment of PDL.

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This work applies deontic operators to actions and many problems with previous deontic logics are avoided.

Two problems with Meyer’s work

Meyer’s work does not handle the normative notions very well, as the following statements are satisfiable in it:

- Killing the president is not allowed, but killing him and then surrendering to the police is.
- Rescuing the injured and then calling an ambulance is obligated, but rescuing the injured is not.

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Its formalization of refraining to do something is not reasonable:

- The intersection of $\overline{\alpha}$ and α is not always empty. This means that there may be ways to refrain from α while at the same time doing α.
- The intersection of $\overline{\alpha}$ and $\alpha;\beta$ is not always empty. This means that performing $\alpha;\beta$ may be a way to refrain from doing α.

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We can not say that if the agent has done a, he has done something else than $a; b$, otherwise we have to say that if he has done $a; b$, he has done something else than $a; b$.

A sharpened idea of reducing normative notions

There are a class of states, a group of people and an agent. Some states are *bad* and some are *fine* for this group. The agent doing an action at a state changes this state to another one.

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An action is *permitted* at a state if the state will always be fine during some performance of this action.

An action is *obligated* at a state if the state will be bad at some point during any performance of *anything else*.

Outline

Language

Let Π_0 be a *finite* set of atomic actions and Φ_0 a *countable* set of atomic propositions.

$$\alpha ::= a \mid 0 \mid (\alpha ; \alpha) \mid (\alpha \cup \alpha) \mid \alpha^*$$
$$\phi ::= p \mid \top \mid \mathfrak{b} \mid \neg\phi \mid (\phi \wedge \phi) \mid \|\alpha\|\phi$$

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$F\alpha$, α is *forbidden*, is defined as $\|\alpha\|\mathfrak{b}$. $P\alpha$, α is *permitted*, is defined as $\langle\!\langle\alpha\rangle\!\rangle\neg\mathfrak{b}$.

Models

$\mathfrak{M} = (W, \{R_a \mid a \in \Pi_0\}, B, V)$ is a model where

- W is a nonempty set of states
- for any $a \in \Pi_0$, $R_a \subseteq W \times W$, and for any $a, b \in \Pi_0$, $R_a \cap R_b = \varnothing$
- $B \subseteq W$
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Roughly a model is a labeled transion system plus a set of bad states.

Interpretation of actions

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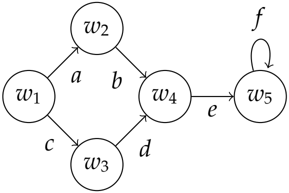

$$
\begin{aligned}
S_{a;b} &= \{w_1 w_2 w_4\}\\
S_{c;d} &= \{w_1 w_3 w_4\}\\
S_{(a;b)\cup(c;d)} &= \{w_1 w_2 w_4, w_1 w_3 w_4\}\\
S_{f*} &= \{w_5, w_5 w_5, w_5 w_5 w_5, \ldots\}\\
&\vdots
\end{aligned}
$$

Semantics

$\mathfrak{M}, w \Vdash \phi$, ϕ being true at w in $\mathfrak{M}$, is defined as follows:

- $\mathfrak{M}, w \Vdash \mathfrak{b} \Leftrightarrow w \in B$
- $\mathfrak{M}, w \Vdash F\alpha \Leftrightarrow$ for any trace $w_0 \dots w_n$, if $w_0 = w$ and $w_0 \dots w_n \in S_\alpha$, then $\mathfrak{M}, w_i \Vdash \mathfrak{b}$ for some $i \leq n$ s.t. $1 \leq i \leq n$
- $\mathfrak{M}, w \Vdash P\alpha \Leftrightarrow$ there is a trace $w_0 \dots w_n$ s.t. $w_0 = w$, $w_0 \dots w_n \in S_\alpha$ and $\mathfrak{M}, w_i \Vdash \neg\mathfrak{b}$ for any $i \leq n$ s.t. $1 \leq i \leq n$

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Computation sequences

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Each action α corresponds to a set $CS(\alpha)$ of seqs.

- $CS(a;b) = \{ab\}$
- $CS((a;b) \cup (c;d)) = \{ab, cd\}$
- $CS(a^*) = \{\epsilon, a, aa, \ldots\}$

The relation of x-different

Let $\sqsubseteq$ denote the relation of *initial segment*. For any seqs σ and τ, σ is *x-different* from τ, $\sigma \not\approx \tau$, if $\sigma \not\sqsubseteq \tau$ and $\tau \not\sqsubseteq \sigma$.

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The relation of x-different formalizes the word "else" in the imperatives such as "don't watch cartoon and do something else". If α is x-different from β, then doing α is doing something else from β.

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Luckily, for any α, among the actions which are something else, there is a *greatest* one in the sense that it is the union of all of them. This lets us deal with the notion of to do something else without introducing any quantifier over actions.

To do something else

Let Δ be a set of seqs. Define $\widetilde{\Delta}$, called the *opposite* of Δ, as the set $\{\tau \mid \tau \not\asymp \sigma \textit{ for any } \sigma \in \Delta\}$.

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Proposition

$CS(\widetilde{\alpha}) = \bigcup\{CS(\beta) \mid \beta \not\approx \alpha\}$.

$\widetilde{\alpha}$ is the union of all the actions x-different from α. To refrain to do α is to do something else; to do *anything else* is to do $\widetilde{\alpha}$.

Obligations

$O\alpha$, α is *obligatory*, is defined as $\|\widetilde{\alpha}\|\mathfrak{b}$; it means that no matter what else except α to do and how to do it, the state will be bad at some point in the process.

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A test $\phi?$ in trace semantics is a set of states in which ϕ is true. Trivially $F(\phi?)$ is never true and $P(\phi?)$ is always true. This means that there is no restriction on testing and testing is always free.

Is there a related algebra?

Let $\mathcal{C}$ be the set of concise sets of seqs. Is there a known algebra related with $(\mathcal{C}, \otimes, \cup, ^*, \widetilde{\ })$?

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Future work

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How do we dynamize it?

Instead of the division of good and evil states, we want to introduce the more fine-grained notion of *betterness* in further work.

Thanks!

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