

Random graphs, first-order logic, and  $AC^0$  circuits.

First-order logic (FO) has very limited expressive power. One of the best-known examples is its 0-1 law on random graphs. Among others, it implies that FO cannot express PARITY. However, once the graphs are ordered, the 0-1 law completely breaks down. Thus, to prove FO cannot define PARITY on ordered graphs, one uses the remarkable machinery of Hastad's Switching Lemma on  $AC^0$  circuits.

In 2008, Rossman proved that the  $k$ -clique problem cannot be defined by FO using only  $k/4$  variables, resolving a long-standing open problem in finite model theory and complexity theory. His proof is built on a brilliant application of Hastad's Switching Lemma on ordered random graphs.

In the talk, I will explain our recent work extending Rossman's result to the so-called planted clique conjecture. Among others, it shows that any super-constant clique cannot be characterized by FO, even in case that the given ordered graphs have a huge planted clique.