What about the next order?

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Workshop on Metamathemics and Metaphysics

For me logic is about definability

Gerald Sacks

For me logic is about what is in the next order

Second-order Lgoic

Second-order logic is a two-sorted logic.

A interpretation of a many-sorted language partition the world into many sorts of entities.

Frege: *Never to lose sight of the distinction between concept and object.* e.g.:

$$\overset{\mathfrak{a}}{-\!\smile\!-}\, F\mathfrak{a}, \quad \text{where } \mathfrak{a} \text{ ranges over objects}$$

today's convention: $\forall x\, Fx$.

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Basic Law V and the Paradox

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$$\varepsilon F = \varepsilon G \leftrightarrow \forall x(Fx \leftrightarrow Gx)$$

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Russell's resolution

No class theory: concepts / properties / functions are not to be correlated with its extension, but are *façon de parler*.

Simple / Ramified type theory:

Entities in the world are partitioned into infinitely many sorts.

Set Theory

Axiomatic set theory can be viewed as an extension of simple type theory to transifinite orders, where the mixture of types is permitted, it provides a framework where elements of higher orders can always be treated as objects, just as the elements of the former orders, namely sets, however, without known contradiction.

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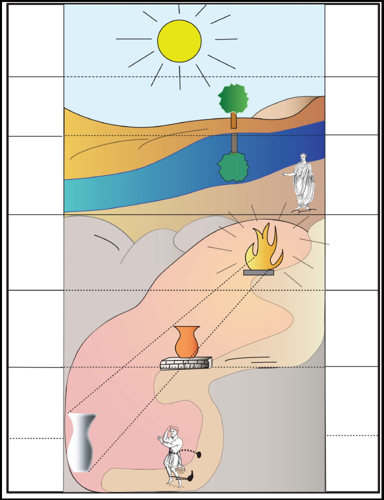

Figure: Allegory of the Cave

Questions

- Is it possible to describe the world in an one-sorted first order language, or is it always inevitable to think of the world in a second-order language?

  Charles Parsons: is whatever is an object?

- Are we making progress when we take a step to the next order? Or are we just moving from ω to $1+\omega$, or taking a loop, or inevitably on a wrong way.

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