Revision 01

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<u>Theorem</u> There is a constant $c_m$. for any $k \in \mathbb{N}$, for any string $\mu$ s.t. $|\mu| \geq 2^{k+c_m+1} + k + c_m + 1$ there is $\delta \prec \mu$ s.t. $C(\delta) < |\delta| - k$

2) For any $d \in \mathbb{N}$, there is $\mu$ s.t. $|\mu| \geq 2^{d+c_{id}+c_m+1} + d + c_{id} + c_m + 1$, (where $c_{id}$ is a coding constant for identity) such that $C(\mu) \geq |\mu|$, and for any such $\mu$, there is $\delta \prec \mu$ s.t. $\mu = \delta\tau$ and

$$C(\mu) > C(\delta) + C(\tau) + d$$

<u>Proof</u> 1) machine $M$: $M(\rho) = \nu\rho$ where $\nu$ codes $|\rho|$

<u>Note</u>: for each $\delta \in \operatorname{ran} M$, $\delta$ is of the form $\nu\rho$ for some $\nu, \rho$.

$$C(\delta) \leq C_M(\delta) + c_m = |\rho| + c_m = |\delta| - |\nu| + c_m$$

Fix $k$, <u>Fix $\mu$</u> s.t. $|\mu| \geq 2^{k+c_m+1} + k + c_m + 1$

Let $\nu = \mu \upharpoonright k+c_m+1$, then $\nu$ codes a number $n \in [2^{k+c_m}, 2^{k+c_m+1})$

Let $\rho = \mu \upharpoonright [k+c_m+1, k+c_m+1+n)$, since $n < 2^{k+c_m+1}$, $\rho$ is well-defined

Let $\delta = \nu\rho$, then $|\delta| < k + c_m + 1 + 2^{k+c_m+1}$, and $\delta \preceq \mu$

Furthermore $C(\delta) \leq |\delta| - |\nu| + c_m = |\delta| - (k+1) < |\delta| - k$

2) Let $k = d + c_{id}$

By counting lemma, there is $\mu$ s.t. $|\mu| \geq 2^{k+c_m+1} + k + c_m + 1$ and $C(\mu) \geq |\mu|$

Fix such a $\mu$. By 1), there is $\delta \prec \mu$ s.t. $C(\delta) < |\delta| - k$

Let $\mu = \delta\tau$. Note $C(\tau) \leq |\tau| + c_{id}$.

Then $C(\mu) \geq |\mu| = |\delta| + |\mu| \geq |\delta| + C(\tau) - c_{id}$

$> C(\delta) + C(\tau) - c_{id} + k = C(\delta) + C(\tau) + d$ □