

12. Universal Martin-Lof test and Solovay test

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Def A Martin-Lof test $\{U_n\}_{n \in \mathbb{N}}$ is universal if for any Martin-Lof test $\{V_n\}_{n \in \mathbb{N}}$, $\bigcap_n V_n \subseteq \bigcap_n U_n$

Therefore if a set A passes a universal Martin-Lof test, it passes all Martin-Lof tests, so it is Martin-Lof-random.

Theorem There is a universal Martin-Lof test

Proof We show that there is an effective enumeration of all M-L tests.

Let $\{R_n^0\}_{n \in \mathbb{N}}$, $\{R_n^1\}_{n \in \mathbb{N}}$, ... be an effective enumeration of all uniformly c.e. subsets of $\mathbb{Z}^{<\omega}$

(We just enumerate the programs)

For each program P_e enumerates $\{R_n^e\}_{n \in \mathbb{N}}$ (note, P_e actually enumerates $\{(n, i) \mid i \in R_n^e\}$)

effectively change it to be Q_e : First enumerates R_n^e uniformly as P_e do once (n, i) try to enter R_n^e , $\mathbb{I}R_n^e\mathbb{I}$ threatens to exceed 2^{-n} stop it.

Let $\{S_n^e\}_{n \in \mathbb{N}}$ be the uniformly c.e. sets enumerated by Q_e

Then $\{S_n^0\}_{n \in \mathbb{N}}$, $\{S_n^1\}_{n \in \mathbb{N}}$, ... is an effective enumeration of all uniformly c.e. sets

and $\mu\mathbb{I}S_n^e\mathbb{I} \leq 2^{-n}$

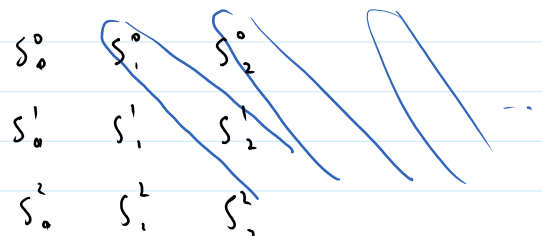
Let $U_n = \bigcup_e \mathbb{I}S_{n+e+1}^e\mathbb{I}$

Then $\{U_n\}_{n \in \mathbb{N}}$ is uniformly c.e.

And $\mu U_n = \sum_e \mu\mathbb{I}S_{n+e+1}^e\mathbb{I}$

$$\leq \sum_e 2^{-(n+e+1)} = 2^{-n}$$

Therefore $\{U_n\}_{n \in \mathbb{N}}$ is a M-L test, and is universal. \square



Note, the M-L test we built:

$$U_k = \{X \mid \exists n \mid (X \upharpoonright n) \leq n-k\}$$
 is already universal.

Corollary The class of 1-random sets has measure 1

Def A Solovay test is a sequence $\{S_n\}_{n \in \mathbb{N}}$ of uniformly c.e. classes ^{of nonempty sets} such that $\sum_n \mu(S_n) < \infty$. A set A is Solovay random if for every Solovay test, A is in only finitely many S_n .

Theorem A set is Solovay random iff it is 1-random

Proof

(\Rightarrow) Note every M-L test is a Solovay test.

So if a set A passes all Solovay tests, it passes all M-L tests

(\Leftarrow) Suppose A is M-L random and $\{S_n\}_{n \in \mathbb{N}}$ is a Solovay test.

We show A passes the test, namely, A is not contained in infinitely many S_n .

Note there is m s.t. $\sum_{n \geq m} \mu(S_n) < 1$

Let $U_k = \left\{ \{ \} \subseteq \mathbb{N} \mid \text{there are } \geq 2^k \text{ many } n > m \text{ s.t. } \{ \} \in S_n \right\}$

Then $\{U_k\}_{k \in \mathbb{N}}$ is uniformly c.e.

$$\text{And } \mu(U_k) \leq 2^{-k} \sum_{n \geq m} \mu(S_n) < 2^{-k}$$

every $\{ \} \subseteq \mathbb{N}$ has been counted at least $\geq 2^k$ many times in $\sum_{n \geq m} \mu(S_n)$

So $\{U_k\}_{k \in \mathbb{N}}$ is a M-L test

So $A \notin U_k$ for some k so A is in at most $\geq 2^k + m$ many S_n \square