II

Mathematical Logic II

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2015

- $f_t(\bar{n}) = t(\bar{n})^{\mathfrak{N}}$ f_t
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- $P \subseteq \mathbb{N}^{k+1}$

$$f(\bar{n}) = \mu m P(\bar{n}, m)$$

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- $P \subseteq \mathbb{N}^{k+1}$

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$$g : \mathbb{N}^k \to \mathbb{N} \qquad h : \mathbb{N}^{k+2} \to \mathbb{N}$$

$$f(\bar{m}, 0) = g(\bar{m})$$

$$f(\bar{m}, n+1) = h(\bar{m}, n, f(\bar{m}, n))$$

f

Q E

(E1) $xE0 = 1$

(E2) $xESy = xEy \cdot x$

Q_E

$\langle a_0, \ldots, a_n \rangle$ $(x)_y$ Q_E

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Q_E

$\langle a_0, \ldots, a_n \rangle$ $(x)_y$ Q_E

$$f(\bar{m}, n) = \left(\mu s\Big[(s)_0 = g(\bar{m}) \wedge \forall x < n (s)_{x+1} = h(\bar{m}, x, h(\bar{m}, x))\Big]\right)_n$$

$(x)_y, g$ h

f Q_E

Q Q

$$f(\bar{m}, n) = (\mu s[(s)_0 = g(\bar{m}) \wedge \forall x < n (s)_{x+1} = h(\bar{m}, x, h(\bar{m}, x))])_n$$

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f Q_E

Q Q

Q $\beta : \mathbb{N}^2 \to \mathbb{N}$

$(a_0, \ldots, a_n)$ a $\beta(a, i) = a_i$

$i \leq n$

(Bézout's identity)

a, b u v $ua + vb = 1$

Proof.

- a, b d

  u, v $ua + vb = d$

- Extended Euclidean algorithm

( )

$d_0, \ldots, d_n \in \mathbb{N}^+$

$a_0, \ldots, a_n \in \mathbb{N}$ $a_i < d_i$ $0 \leq i \leq n$

$$x \equiv_{d_0} a_0$$

$$x \equiv_{d_1} a_1$$

$$\vdots$$

$$x \equiv_{d_n} a_n$$

$s \in \mathbb{N}$ $s+1$

$1+1 \cdot s!$ $1+2 \cdot s!$ ...... $1+(s+1) \cdot s!$

Proof.

$1+(i+1) \cdot s!$ $1+(j+1) \cdot s!$ >1 d

$$\alpha : \mathbb{N}^3 \to \mathbb{N} \qquad \alpha(c, d, i) = \mathrm{rem}(c, 1 + (i + 1)d)$$

α

Proof.

$\alpha(c, d, i) = \mu r\, [\exists q \le c(c = q(1 + (i + 1)d) + r)]$

$$\alpha : \mathbb{N}^3 \to \mathbb{N} \qquad \alpha(c, d, i) = \mathrm{rem}(c, 1 + (i + 1)d)$$

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Proof.

$\alpha(c, d, i) = \mu r\,[\exists q \le c(c = q(1 + (i + 1)d) + r)]$

$$J(a,b) = \frac{1}{2}(a+b)(a+b+1) + a$$

$$K(p) = \mu a \leq p \; \exists b \leq p \; J(a,b) = p$$

$$L(p) = \mu b \leq p \; \exists a \leq p \; J(a,b) = p$$

β $\beta(s,i) = \alpha(K(s), L(s), i)$ $\beta(s,i)$

Q $n, a_0, \ldots, a_n$

s $i \leq n$ $\beta(s,i) = a_i$

$\beta(s,i)$

$(x)_y$

β $\beta(s, i) = \alpha(K(s), L(s), i)$ $\beta(s, i)$

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$\beta(s, i)$

$(x)_y$

Q

Proof.

g, h f g, h

$$F(\bar{m}, n) = \mu s\Big[\beta(s, 0) = g(\bar{m}) \wedge \forall i < n\big(\beta(s, i+1) = h(\bar{m}, i, \beta(s, i))\big)\Big]$$

F $f(\bar{m}, n) = \beta(F(m, n), n)$ f

Q

Proof.

g, h f g, h

$F(\bar{m}, n)$

$$= \mu s \Big[\beta(s, 0) = g(\bar{m}) \wedge \forall i < n \big(\beta(s, i+1) = h(\bar{m}, i, \beta(s, i)) \big) \Big]$$

F $f(\bar{m}, n) = \beta(F(m, n), n)$ f

T Q

- $f : \mathbb{N}^k \to \mathbb{N}$ f f T

- $R \subseteq \mathbb{N}^k$ R R T