- 0
- c_k^n
- $\mathrm{pred}(x)$

$\dot{-}$ quo rem

- 0
- c_k^n
- $\mathrm{pred}(x)$

$\dot{-}$ quo rem

- 0
- c^n_k
- $\mathrm{pred}(x)$

  $\dot{-}$ quo rem

- 0
- c_k^n
- $\mathrm{pred}(x)$

$\dot{-}$ quo rem

- 0
- c_k^n
- $\mathrm{pred}(x)$

$\dot{-}$ quo rem

- σ δ
- n p_n
- lh $(a)_i$

  $a^\frown b$

/ /

- $f(\bar{x})$ $\left\{(\bar{x}, y) \;\middle|\; f(\bar{x}) = y\right\}$
-
-

$f(y+1)$

$f(y)$

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$f(y+1)$

$f(y)$

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$($ $)$

$f(\bar{x}, y)$ $k+1$

$$F(\bar{x}, n) = \langle f(\bar{x}, 0), \ldots, f(\bar{x}, n) \rangle$$

k g $k+2$ h $k+1$ f g

h

$$f(\bar{x}, 0) = g(\bar{x})$$

$$f(\bar{x}, y+1) = h(\bar{x}, y, F(\bar{x}, y))$$

k g $k+2$ h $k+1$ f g

h

$$f(\bar{x}, 0) = g(\bar{x})$$

$$f(\bar{x}, y+1) = h(\bar{x}, y, F(\bar{x}, y))$$

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Ackermann function

$$A(0, y) = y + 1$$

$$A(x + 1, 0) = A(x, 1)$$

$$A(x + 1, y + 1) = A(x, A(x + 1, y))$$

$$A(1, y) = y + 2$$

$$A(2, y) = 2y + 3$$

$$A(3, y) = 2^{y+3} - 3$$

$$\cdots\cdots$$

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(μ)

f $k+1$ g f

μ $\bar{x} \in \mathbb{N}^k$ $y \in \mathbb{N}$

$f(\bar{x}, y) = 0$ $g(\bar{x}) =$

$f(\bar{x}, y) = 0$ y $g(\bar{x}) = \mu y[f(\bar{x}, y) = 0]$

$g = \mu_r(f)$

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$$g = \mu_r(f)$$

- f $g = \mu_{\mathrm{r}}(f)$
- f g

- f $g = \mu_{\mathrm{r}}(f)$
- f g

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$k+1$ P $\bar{x} \in \mathbb{N}^k$

$y \in \mathbb{N}$ $P(\bar{x}, y)$

$$g(\bar{x}) = \mu y P(\bar{x}, y)$$

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$k+1$ P $\bar{x} \in \mathbb{N}^k$

$y \in \mathbb{N}$ $P(\bar{x}, y)$

$$g(\bar{x}) = \mu y P(\bar{x}, y)$$

Proof.

$S \subset \mathbb{N}^3$

- $(0, y, z) \in S$ $z = y + 1$
- $(x+1, 0, z) \in S$ $(x, 1, z) \in S$
- $(x+1, y+1, z) \in S$ u $(x, u, z) \in S$

$(x+1, y, u) \in S$

Proof.

$S \subset \mathbb{N}^3$

- $(0, y, z) \in S$ $z = y + 1$
- $(x + 1, 0, z) \in S$ $(x, 1, z) \in S$
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$(x + 1, y, u) \in S$

7.2.1 7.2.2

f

-
- g $\bar{x}$ $f(\bar{x}) \leq g(\bar{x})$

$x, y \in \mathbb{N}$ Nice S

$(x, y) \in \operatorname{dom} S$